



## Midterm 1 Finance I

### 1. Key concepts

- a. We know that an asset's risk is the sum of two different types of risk, diversifiable (or specific risk) and non-diversifiable (or market risk). What type of risk should we consider when determining the cost of opportunity and why? (10 pts)

We should only consider non diversifiable risk, because diversifiable risk can be eliminated (diversified) by holding the market portfolio and therefore is not required to provide an expected return for that risk.

(3 points for choosing the right answer, 7 points for the explanation)

- b. Why the risk of a portfolio can't be higher than the weighted average risk of its assets'? (10pts)  
Include a mathematic proof.

Because in the event of perfect positive correlation, the risk of a portfolio equals the weighted average risk of its assets'. There is no higher correlation than perfect; therefore it's impossible for a portfolio to have greater risk. Mathematically, using the variance formula ( $x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2\rho$ ), we can see that the correlation coefficient equal to 1 implies the risk of a portfolio equals the weighted average risk of its assets'. Since the correlation coefficient takes values from -1 to 1, it's impossible to reach a risk level superior to the weighted average risk of the portfolio's assets.

(5 points for the explanation conceptual explanation, 5 for the mathematical demonstration)

- c. How does the cost of opportunity of an asset change if the asset's **covariance** increases? Explain your answer (10pts)

The cost of opportunity always increases, as the non diversifiable risk always increases. The increase in covariance may come from 3 sources, increase in the standard deviation, increase in the weight or increase in the correlation coefficient. The first two will impact positively both the variance and covariance and the later will only impact increasing the covariance.

(3 points for choosing the right answer, 7 points for the explanation)



## 2. Portfolio Theory

Suppose you are evaluating an investment in a set of assets. Asset A has an expected return of 18% and a standard deviation of 12%. Asset B has an expected return of 36% and a standard deviation of 20%. The correlation coefficient of both assets is 0.5 (zero point five). The return of a risk free asset is 5%.

- a. Determine what is the weight each asset should have in a portfolio, if you require the portfolio to provide an expected return of 28%. - (10 Pts)

Using the weighted average formula for the expected return, we solve for the weight each asset should have. (Asset A = 44% and Asset B= 56%)

(5 points for using the correct formula, 5 points for answering correctly)

- b. Calculate the variance of the portfolio from 2.a (10 Pts)

Using the formula  $x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2\rho$ , we replace the values from 2.a and obtain a variance equal to 2.11%

(5 points for using the correct formula, 5 points for answering correctly)

- c. Calculate the Sharpe Index of Asset A, Asset B and the portfolio from 2.a. (10 Pts)

Using the formula  $(R_a - R_f) / \sigma_a$  we calculate each index (or ratio). Keep in mind that the value of the denominator is the standard deviation and the value calculated in 2.b is the variance, therefore you must use the square root of that value

Sharpe A:  $(18\% - 5\%) / 12\% = 1.08$

Sharpe B:  $(36\% - 5\%) / 20\% = 1.55$

Sharpe Portfolio:  $(28\% - 5\%) / 14.5\% = 1.58$

(5 points for using the correct formula, 5 points for answering correctly)



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- d. Assuming you are risk averse, (but not so averse as to hold only the risk free asset).- Determine what investment could maximize your investment possibilities (A, B or the portfolio from 2.a) Please explain. You may include a graph to support your explanation (10 Pts)

The graph is not necessary, but if included it will yield 3 points (if score from answer is less than 10) We always choose a combination with the asset or portfolio that holds the highest Sharpe Index, therefore we choose the portfolio over A and B. This results on higher expected returns for any level of risk.

(5 points for choosing the right answer, 5 points for the explanation)

### 3. CAPM

Suppose the risk free rate is 4% and the market's expected return is 11%. The market portfolio has a standard deviation of 25%. Assets C and D have betas of 0.7 and 1.1 respectively

- a. What is the appropriate cost of opportunity according to CAPM for assets C and D (10pts)

Using the CAPM formula:

$$R_c = r_f + B_c \cdot (R_m - R_f)$$

$$R_c = 4\% + 0.7 \cdot (11\% - 4\%) = 8.9\%$$

$$R_d = r_f + B_d \cdot (R_m - R_f)$$

$$R_d = 4\% + 1.1 \cdot (11\% - 4\%) = 11.7\%$$

(5 points for using the correct formula, 5 points for answering correctly)

- b. What is the appropriate cost of opportunity according to CAPM for a portfolio holding 70% of asset C and 30% of asset D (10pts)

Two possible ways: use the weighted average of the cost of opportunity:

$$70\% \cdot 8.9\% + 0.3 \cdot 11.7\% = 9.74\%$$

Or using the weighted average of betas and replacing in the CAPM formula:

$$70\% \cdot 0.7 + 0.3 \cdot 1.1 = 0.82$$

$$R_p = r_f + B_p \cdot (R_m - R_f)$$

$$R_p = 4\% + 0.82 \cdot (11\% - 4\%) = 9.74\%$$



(5 points for using the correct formula, 5 points for answering correctly)

- c. Suppose there is an additional Asset E, with a beta of -0.7 (negative zero point seven). How does its cost of opportunity (according to CAPM) compare to the cost of opportunity of Asset C? Does this mean that Asset E is less risky?(10 pts)

The cost of opportunity is calculated using CAPM

$$R_e = r_f + \beta_e \cdot (R_m - R_f)$$

$$R_e = 4\% - 0.7 \cdot (11\% - 4\%) = -0.9\%$$

The cost of opportunity is lower, since the correlation with the market portfolio is negative. This doesn't mean it's less risky. It's actually just as risky, as it varies just as much as asset C (70% of the market risk), however, it's risk behavior is countercyclical to the market's, therefore for a diversified investor (which according to the theory, we are all diversified investors) it serves as insurance against market decrease.

(5 points for using the correct formula, 5 points for answering correctly)